# The Impact of Inventory Risk on Market Prices Under Competition 

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#### Abstract

: Firms often must procure inventory/capacity before knowing what the demand will be, so there is a potential for a mismatch between inventory and demand, the "inventory risk." We show that because of inventory risk, an increase in the number of competitors can lead to an increasing trend in market prices. Furthermore, we show that, ceteris paribus, because of how inventory risk impacts competitive behavior, firms may prefer to incur inventory risk rather than to avoid it. We establish these findings using three rather different yet complementary methodologies: (i) using data from a classroom experiment, (ii) using a quantal response equilibrium simulation to capture realistic irrationalities in managerial decisions, and (iii) using a fully-rational Nash equilibrium model. That three very different methods lead to identical qualitative findings reinforces the main message of our paper: inventory risk reverses the standard intuition for how an increase in the number of competitors impacts prices.


Keywords: competition; inventory risk; pricing;

## 1. Introduction

Standard economic theory suggests that an increase in the number of competitors in a market should lead to a decrease in market prices and profits. But the standard theory disregards an important operational element of how firms compete: to be able to meet demand, firms often must procure inventory (build capacity, secure supply, etc.) in advance of knowing what the actual demand will be, and a mismatch between inventory/capacity and demand can be costly. In this paper, we show that accounting for this mismatch—which we refer to as the inventory risk-may reverse the logic of the standard theory, so that an increase in the number of competitors may, in many cases, lead to an increasing trend in average market prices, and can, as a result, also increase profits above those obtainable without inventory risk.

The idea that inventory risk may impact prices this way was suggested to us by MBA and Executive MBA students who played a classroom supply chain simulation called "The Supply Chain Game" (See Appendix C for a detailed description), - a competitive extension of the classical Beer Game (Sterman 1989). We therefore first describe the game and the observations from it.

### 1.1 Observed Pricing in the Supply Chain Game

The Beer Game considers a serial supply chain consisting of a retailer, a wholesaler, a distributor, and a brewery, and exposes students to challenges of managing inventory and information among independent decision makers. The Supply Chain Simulation Game (Appendix C) extends this setup by considering multiple Beer-Game-like supply chains in which retailers compete for consumer demand and, in addition to quantities, also decide on the retail prices. The game is usually played for 8 quarters, each with 12 weeks / periods, such that traditional "beer game" quantity decisions are made every week, while pricing decisions are made every quarter ${ }^{1}$. When deciding their retail price, retailers had to consider the demand in the market which, as in most practical situations, included two types of consumers: some consumers were loyal to their retailer / "brand" of choice, while others were not loyal and were willing to shop

[^0]around for the retailer with the lowest price; we will call them "bargain-hunters" later in the paper. See Appendix C for more information on what the students knew about the demand in the game.

We played this game with graduate (MBA) students and business executives (EMBA students), across North America and abroad, in both the core operations management classes (for EMBA students) and in supply chain management electives (for full time MBA students). The game was played as a practical exercise and a capstone activity close to the end of the respective courses. Depending on the number of students, we varied the number of teams from three to eight (we never played with two teams and rarely played with more than six, for pedagogical reasons). As mentioned, the games progressed for five to eight quarters, depending on the time allotted in the classes, with pricing decision made by every retailer once per quarter. As a result, across all games played, we observed 192 pricing decisions with a mean price of 123 and a standard deviation of 73.76 . Table 1 provides the results of regressing the observed prices on the number of competing retailers, controlling for the quarter and the type of the course (prices tend to increase with participants' experience with the game and are generally lower in the supply chain management elective.)

Table 1: Observed prices in the Supply Chain game, 2009-16. $n=192$, adjusted $r^{2}=0.25$

|  | Coefficients | Standard Error | t-Stat | P-value |
| :--- | :---: | :---: | :---: | :---: |
| Intercept | -50.988 | 27.417 | -1.860 | 0.064 |
| \# of competitors | 38.639 | 6.151 | 6.282 | 0.000 |
| Quarter | 8.659 | 2.215 | 3.909 | 0.000 |
| SCM elective | -39.530 | 9.417 | -4.198 | 0.000 |

The immediate observation is that, on average, the observed prices increase significantly with the number of competing retailers. Such a price increase initially made no sense to us, as it directly contradicts the common logic and predictions of the standard theory, but its persistent recurrence with both full-time students and experienced business executives motivated us to study the phenomenon further.

### 1.2 Real Life Example: Target’s Canadian "Misadventure"

Before proceeding with the model development, we note that the connection between the firm's pricing, inventory risk and profits is not just a classroom game phenomenon.

In March 2013, Target, the then second largest U.S. retailer (after Walmart) entered the Canadian market. Despite being perceived in its domestic (U.S.) market as a premium brand ${ }^{2}$ when compared to Walmart, Target entered Canada with a low-price strategy (Shaw 2013), undercutting Walmart, especially in the non-grocery categories ${ }^{3}$. However, it appears that Target massively underestimated the demand generated by their low-price strategy and suffered from widespread inventory shortages. The initial consumer excitement quickly changed to disappointment from "bare shelves" and "unacceptable" out-ofstocks, forcing Target to close all its 124 stores in Canada and exit the market (Shaw 2014) with multibillion losses for the company, and job losses for the leadership team responsible for Target's first-ever international expansion (including the then group CFO and several other top executives).

Target's Canadian "misadventure" has a direct parallel to the Supply Chain game described earlier. While the total market demand was uncertain, it is unlikely that the increase in Target's demand was caused by a change in the overall market demand-rather, this demand surge was likely caused by consumers switching from Walmart (and other players) to Target in response to the low-price strategy. Our paper argues that the existence of such highly volatile "bargain-hunting" consumers makes Target's inventory management much more challenging under low-price strategy because of the possible large disconnect between demand and supply-that is, inventory risk. This link between pricing, inventory risk, and the difficulty of the inventory management is overlooked in the standard economic theory that predicts that price decreases as new competitors enter, and our paper is the first to show that this qualitative prediction may be reversed. Translating back to the Target story, our paper suggests that the retail giant might have been better off entering Canada with a premium pricing strategy because doing so would have resulted in less inventory risk and allowed them to better deliver on their customer value proposition. This qualitative conclusion is also consistent with the common rationale presented by the students during the Supply Chain game debrief.

[^1]
### 1.3 Our Approach and Contributions

We consider a market in which symmetric competing firms determine their prices and quantities/inventories before knowing other firms’ decisions and demand realizations, while facing two types of costs: inventory/procurement cost and shortage cost. The market consists of two types of consumers: loyal consumers, who are spread equally across all firms and purchase the product depending on the price at their firm of choice; and bargain-hunting consumers, who switch to a firm with the lowest price. That is, by charging a relatively high price the firm targets only its own loyal consumers, but by charging a relatively low price it also competes for the bargain-hunters. In addition, firms procure inventory; how much inventory to procure clearly depends on the pricing decision. In equilibrium, firms choose prices and quantities, and we study how average prices and profits depend on the number of firms in the market.

Within this general framework, which matches the Supply Chain game, and, we believe, is reflective of multiple practical situations, we present two kinds of models. First is the Quantal Response Equilibrium, QRE, model (Section 4) with boundedly-rational competitors - indeed, the students in the game, as well as practicing managers, exhibit irrationalities in their decision making, and the numerical simulations with our QRE model capture them. Second is the Nash Equilibrium model (Section 5), where we provide analytical proof for the emergence of the price-increasing trend even with full rationality.

Our model generates two main insights. First and foremost, we show that an increase in the number of competitors may indeed lead to an increasing trend in average market price. The intuition behind this phenomenon is as follows. Firms that follow a high-price strategy need to order little inventory (for their loyal customers only), while those with a low-price strategy need to order a lot (for their own loyals and for all the bargain-hunters). The challenge is that, if a firm charges a low price and orders a lot of inventory but does not win the bargain hunters' demand because some other firm charges an even lower price, it will end up with large leftovers. In the reverse scenario, if the firm orders little inventory but is visited by bargain hunters, it will face large inventory shortages. Both kinds of mismatch are costly in our model (as they are in practice), and thus, in equilibrium, many firms choose to err on the
high-price side so that even if the mismatch occurs, they have a high margin to absorb it. The more competitors there are in the market, the less certain each of the firms is of winning the bargain-hunters' demand, which also becomes proportionally larger in comparison to the retailer's own loyal demand. In other words, the inventory risk increases with the number of competitors. As a result, in situations with more competitors, more firms choose the high-price strategy and the prices increase on average. This matches with Target's story and the observations from the Supply Chain game; as mentioned above, the intuition also aligns, in broad strokes, with the statements participants made during the game debrief. Furthermore, such a price-increase phenomenon is observed in both boundedly-rational and fully-rational models, which gives us additional confidence that the observations from the Supply Chain game were not driven merely by the irrationality in the students' behaviors.

The above discussion, however, is incomplete because it demonstrates correlation and not causality, and thus our analysis does not stop there. Specifically, while the above argument implies that a price-increase phenomenon appears in equilibrium in the presence of inventory risk, it does not necessarily suggest that it happens because of inventory risk. To prove the latter, clearly stronger, claim, we also show that, once the inventory risk is removed, the average market price exhibits a decreasing trend in equilibrium. To do so we consider two variants of our Nash Equilibrium model. In Section 5.1 the firms manufacture to stock (MTS); inventory risk is present, and we show that the resultant equilibrium prices increase, on average. In Section 5.2 the firms manufacture to order (MTO), meaning they face no inventory risk as they can produce the necessary quantity after the demand realizes, and the resultant prices decrease, on average. That is, once the inventory risk is removed, we observe the "intuitive" price-decreasing trend. This establishes causal connection between the inventory risk and market prices and emphasizes our main result.

Our second finding is that while inventory risk exists in the MTS case, but not in the MTO one, we show that firms may, nevertheless, prefer to operate under the former and incur the inventory risk rather than avoid it. This is because of two counteracting effects. On the one hand, firms certainly would want to avoid the inventory risk. On the other hand, as we show, exposure to inventory risk softens
competition and the benefit from that could, under certain conditions, exceed the costs from the mismatch between supply and demand. This result does not necessarily imply that firms would be disinterested in building more responsive operations. Rather, the implication is that of the well-known prisoners' dilemma: the industry as a whole (i.e., the "average firm") might prefer the MTS equilibrium with softer competition, while individual firms might deviate, spiraling-down to an MTO equilibrium with lower profits due to the intensified competition.

Combining these insights, the key message of our paper is that inventory risk impacts firms’ competitive strategy in non-trivial ways, with primary implications being the possible increase in prices, the softening of competition, and an increase in profits. These implications are important for both business executives who decide on which markets to enter and how, as well as for government officials who decide on measures for promoting competition. Specifically, for the latter, our results imply that governments need to encourage firms to have more responsive supply chains, which could not only reduce inventory inefficiency, but also intensify competition, leading to lower prices.

## 2. Literature Review

Our paper draws on four streams of literature: operations management studies of (i) inventory and pricing competition, (ii) multi-method studies of inventory management and inventory risk, (iii) MTS versus MTO systems, and (iv) the economics studies of price-increasing competition.

Much of the operations management/research literature focuses exclusively on inventory competition (e.g., Lee and Lu 2015, Li and Ha 2008, Lippman and McCardle 1997, Mahajan and van Ryzin 2001, Nettesine and Rudi 2003, and Parlar 1988). These papers provide useful modeling constructs that we build upon, but offer no direct guidance for our research question as they treat price as being given exogenously. In contrast, by endogenizing the pricing decision, we extend, and, in some cases, reverse their insights. For instance, Mahajan and van Ryzin (2001) show that, as the number of competing firms increases, firms overstock so much that the total system profit approaches zero. We, however, show
that only a few firms charge the low price (and overstock) in equilibrium so that the total system profit increases as the proportion of low price/overstocking firms decreases with the increase in the number of competitors.
(i) Firms in our model decide on quantities before the uncertain demand realizes, and in this sense they are similar to the classical "newsvendors." The monopolistic price-setting newsvendor model is well researched (e.g., Aydin and Porteus 2008, Dong et al. 2017, Karlin and Carr 1962, Kazaz and Webster 2015, Mills 1959, Petruzzi and Dada 1999, and Raz and Porteus 2006), but competition on both price and quantity has proven to be a notoriously difficult problem. Bernstein and Federgruen (2004) show the existence of the equilibrium in the game of joint pricing and service level competition. Parlar and Weng (2006) consider the coordination mechanism between two firms that competes on pricing and quantity. However, these two papers do not study how the properties of this equilibrium change with the number of firms. Zhao and Atkins (2008) hint at the possibility of seeing a result like ours, but because their research question is quite different, their model cannot be directly extended to capture our phenomenon. Cho and Wang (2017) examine the impacts of collusion, pooling, and synergy derived from the merger of pricesetting newsvendors; the key driver of their result comes from the savings in inventory costs which has some similarity to "easing" of inventory management when the number of competitors is reduced in our paper. Despite this similarity, their research question is quite different, and their work does not offer direct insights onto how prices change with the number of competitors. Finally, Chen et al. (2004) also examine an industry where multiple competing firms make pricing and inventory decisions. They show that a ratio between the equilibrium price and the cooperative price decreases in the number of competitors. However, carefully reconstructing their example shows that the resultant equilibrium price can increase in the number of firms, because the cooperative price increases at a rate faster than the ratio decreases. Hence, their result provides additional anecdotal support for our insights and reinforces their generalizability.
(ii) Our paper is also related to the literature that uses multiple methods to study inventory management and inventory risk. Ketzenberg et al. (2000) consider a dense retail outlet that must balance
between high product varieties while minimizing the possibility of stocking out. They propose a heuristic for deciding the inventory to order, and then compare how this heuristic performs against the current practice and the optimal solution. Closs et al. (2010) examine the relationship between inventory, product complexity and demand uncertainty for a configure-to-order system using a simulation model, and demonstrate the different impacts of these variables to unit and order fill rates. Bicer et al. (2018) and Udenio et al. (2018) demonstrate the value of inventory flexibility in the presence of demand uncertainty and examine the value of reducing lead time under various scenarios; this essentially reduces inventory risk. Along the same line, Udenio et al. (2018) measure the value of inventory agility when the stocked inventory differs from the forecasted value; again, a concept related to inventory risk.
(iii) Considering the studies of the MTS versus MTO systems, (see Gunasekaran and Ngai 2005, 2009 for extensive reviews), several authors discuss how these systems are impacted by competition. Caro and Martínez-de-Albéniz (2010) show that an asymmetric (MTO versus MTS) outcome can be desirable for both competitors; our results agree with theirs as in equilibrium some firms charge the high price (and target only their loyal customers), while others charge the low price and compete for bargainhunters. Sun et al. (2008) show that, under asymmetric competition, the MTO firm will charge a lower price than the MTS one to compensate for the delay in product delivery. Our finding that prices will be lower under MTO directionally agrees with theirs, but is driven by the intensified competition and not delays.
(iv) In the economics literature, the main phenomenon of our work is known as "price-increasing competition." Rosenthal (1980) demonstrates this effect when new entrants increase market size (as a side note, market size also increases in Chen et al. 2004). Anderson et al. (1995) and Chen and Riordan (2008) show that a price increase can be driven by product differentiation. However, neither of these papers considers inventories; thus, our work offers a novel explanation for a possible price-increase that is particularly salient in operational settings: inventory risk.

## 3. Model Primitives

Consider an industry with $N \geq 2$ firms selling identical products with a unit production/procurement cost $c$. Each firm, $i \in\{1, \ldots, N\}$, commits to a selling price, $p_{i}$, and an order quantity/inventory/capacity/etc. ${ }^{4}$, $Q_{i}$, before demand is realized ${ }^{5}$ and before it learns the decisions of the other firms. The market demand consists of a mixture of two types of customers: a fraction $\lambda \in(0,1)$ of loyal customers, and a fraction ( $1-\lambda$ ) of bargain-hunters. Loyal customers are spread equally across all firms such that firm- $i$ demand from loyal customers is $D^{L}\left(p_{i}\right)=\frac{\lambda}{N}\left(a-b p_{i}\right)$; importantly, because, as their name suggests, loyal customers do not consider other firms, loyal demand is independent of the prices charged by other firms. In contrast, bargain-hunters, as their name suggests, search for the firm with the lowest price and purchase from it, leading to the following demand function:

$$
D^{B}\left(p_{i}, \boldsymbol{p}_{-i}\right)=\left\{\begin{array}{cc}
(1-\lambda)\left(a-b p_{i}\right) & \text { if } p_{i}<\min \left[\boldsymbol{p}_{-i}\right]  \tag{1}\\
(1-\lambda)\left(a-b p_{i}\right) \text { w. prob. } 1 /\|k\| & \text { if } \exists\{k\} \text { s.t. } p_{i}=p_{j}, j \in\{k\}, \\
0 & \text { otherwise }
\end{array}\right.
$$

where the bold $\boldsymbol{p}_{-\boldsymbol{i}}$ refers to a vector of prices of all firms other than $i$. Let $D_{i}\left(p_{i}, \boldsymbol{p}_{-\boldsymbol{i}}\right)=D^{L}\left(p_{i}\right)+$ $D^{B}\left(p_{i}, \boldsymbol{p}_{-\boldsymbol{i}}\right)$ denote the total demand of firm $i$ when it charges price $p_{i}$ and other firms charge $\boldsymbol{p}_{-\boldsymbol{i}}$.

Each firm incurs a shortage cost $s$ per unit of unsatisfied demand. Therefore, when the realized demand is $D_{i}$ and firm $i$ sets price $p_{i}$ and orders quantity $Q_{i}$, it would receive the following profit:

$$
\begin{equation*}
\pi_{i}\left(p_{i}, q_{i}, \boldsymbol{p}_{-i}\right)=p_{i} \min \left[D_{i}\left(p_{i}, \boldsymbol{p}_{-i}\right), Q_{i}\right]-c Q_{i}-s\left[D_{i}\left(p_{i}, \boldsymbol{p}_{-i}\right)-Q_{i}\right]^{+} \tag{2}
\end{equation*}
$$

Our analyses consider the equilibrium $p_{i}$ and $Q_{i}$ which emerge as firms maximize the expectation of the profit in (2) over the demand uncertainty induced by the actions of other firms. Before proceeding further, we make two important remarks:

[^2]Remark 1 ("winner-takes-all" allocation of bargain-hunting demand): An implicit assumption in $D^{B}$ is that all bargain-hunting demand is allocated to a random lowest-price firm. In practice, however, it is reasonable to expect that all such firms would capture some fraction of the bargain-hunting demand. Assuming that this demand will be split equally, though, is unrealistic; factors such as word-of-mouth and popularity-based rankings in search engines are known to result in situations where sellers of similar "quality" products end up having dramatically different market shares, typically with one attracting the majority of demand and the rest obtaining only a small share (e.g., see the highly-cited Science Magazine article by Salganik et al. (2006) or the book by Frank and Cook (1995)). Appendix B. 1 shows that our insights extend to such a case as well, meaning that the "winner-takes-all" assumption is not restrictive.

Remark 2 (uncertain vs deterministic total market demand): another implicit assumption is that the total market demand is deterministic, and the demand uncertainty comes from the behavior of bargainhunters and the decisions of other firms. In practice, both market and competition-induced uncertainties are clearly present, however: (i) the Target story hints that the inventory risk is largely driven by the latter; and (ii) we ran a numerical simulation with uncertain market demand and noticed that it had no qualitative impact on the observed phenomenon.

Next, we analyze the impact of inventory risk on prices in the equilibrium. In particular, as argued in the debrief of the Supply Chain game, two kinds of factors impact the behavior of a firm in a competitive situation: fully rational profit maximizing factors, and boundedly rational, behavioral factors of the firm's own managers, and those of their competitors. We discuss both next.

## 4. [Boundedly-Rational] Quantal Response Equilibrium Model

Our approach to capturing bounded rationality relies on the observation that even when making standard inventory ordering decisions (in isolation or in competition) repetitively in an unchanged circumstances, human decision makers routinely changed their decisions, while the rational action was to order the same (optimal) quantity every time; e.g., see Schweitzer and Cachon (2000), Bolton and Katok (2008), Bolton et al. (2012) for the isolated case, and Ovchinnikov et al. (2015), Feng and Zhang (2017), Quiroga et al.
(2019) for the competitive case; see also Zhang and Siemsen (2019) for meta-analyses. That is, decision makers are boundedly rational, and we capture that through the concept of quantal response equilibrium (QRE) introduced by McKelvey and Palfrey (1995). In particular, we use the logit QRE model, in which the distribution $A_{i}$ according to which player $i$ chooses action pair ( $p_{i}, q_{i}$ ) given the opponents' distributions $A_{-i}$ is given by the following probability function:

$$
\operatorname{Prob}_{i}\left(p_{i}, q_{i} \mid A_{i}, A_{-i}\right)=\frac{\exp \left[E_{A_{-i}}\left\{\pi_{i}\left(p_{i}, q_{i}, \boldsymbol{p}_{-i}\right) / \beta\right\}\right]}{\sum_{\hat{p}_{i}, \hat{q}_{i} \in A_{i}} \exp \left[E_{A_{-i}}\left\{\pi_{i}\left(\hat{p}_{i}, \hat{q}_{i}, \boldsymbol{p}_{-i}\right) / \beta\right\}\right]}
$$

where $\beta$ is the (ir)rationality parameter.
Our formulation is consistent with the idea of attraction models, e.g., Camerer and Ho (1999), in the sense that the actions that lead to higher payoffs will be chosen with larger probabilities. It is easy to see that when $\beta$ is large, the distribution will be more "flat"; in the limit when $\beta \rightarrow \infty$, it will converge to a uniform distribution over all possible actions/strategy pairs ( $p_{i}, q_{i}$ ) - that is, players who exhibit little rationality make random decisions regardless of the resultant profits. But when $\beta$ is small, in the limit when $\beta \downarrow 0$, the QRE distribution will converge to the profit-maximizing choice made with probability 1 , a case with full rationality. See Chen et al. (2012) for detailed discussion of the use of QRE logic in the models of bounded rationality, as well as Wu and Chen (2014) for an application to inventory decisions.

QRE models are generally not amenable to analytical investigations, and we therefore perform a series of numerical simulations to understand how the average price might change if the number of competitors changes for $N=2,3,4$. For the "base case" we assume the following parameter values: rationality parameter, $\beta=50$, market size, $a=500$, demand ${ }^{6}$ slope parameter $b=100$, fraction of loyal customers, $\lambda=0.5$, purchase cost, $c=1$, shortage cost, $s=0.5$. To implement QRE we discretized the firm's prices such that $p_{i,-i} \in\{1,1.5, \ldots, 5\}$ and we used the resultant $q_{i,-i} \in\{0,50, \ldots, 400\}$ for quantities. Following earlier QRE implementations, such as Chen et al. (2012), Wu and Chen (2014), we looked

[^3]only for symmetric equilibria. Computations were performed in Wolfram Mathematica on a multi-core desktop PC; the code is available upon request.


Figure 1: The average equilibrium price (a), quantity (b), and profit (c), for $N=2,3,4$.

Figures 1a and 1b present the firm's average equilibrium price and quantity. First and foremost, as the number of competitors grows from $N=2$ to $N=4$, the price increases from approximately $\$ 2.50$ to $\$ 2.80$, - the key phenomenon we study in this paper. At the same time, the quantity decreases from approximately 165 to 85 . In all cases, however, there is an excess of inventory: indeed the total market demand at the price of 2.5 is $500-100 \times 2.5=250$, while the inventory for two firms, for example, is $165 \times 2=330>250$. This is because the possibility that a firm may capture bargain-hunting demand increases the amount of inventory it procures; we discuss this in more detail later. However, while each firm's quantity decreases, the total quantity over all firms slightly increases from approximately 330 to 340. This is because of the opposing influence of two effects: a higher price decreases the mean demand, which, intuitively, decreases the order quantity. However, a higher price also increases the underage cost, which increases the optimal critical fractile and order quantity. The second effect can dominate in certain cases, Raz and Porteus (2006), as it does here. Zhao and Atkins (2008) present a conceptually similar result too: in their model the equilibrium price may increase vis-a-vis the monopoly price (they did not vary the number of competitors) and the total quantity may increase.

Figures 1c shows that the average profit is decreasing and with $N=4$ the expected profit is negative. This implies that the equilibrium with four competitors is not sustainable; in a practical
situation, one of the firms would exit the market, restoring a positive-profit equilibrium with three firms. Consequently, there is no need to consider the case with $N=5$ or more firms given our model parameters. [Remark: In the Nash model in Section 5 we consider situations with more firms.]

To see how inventory risk impacts this counterintuitive price-increasing equilibrium, consider Figure 2. For $N=2$ (left) and $N=4$ (right) it presents four quantities as a function of the firm's price, $p_{i}$ : the expected equilibrium demand (solid line), the firm's demand if its price is not lowest (firm's own loyal demand, lower error bars), the firm's total demand if the price is lowest (own loyals + all bargain-hunters, upper error bars), and the probability of the firm's price being lowest (dotted line, right axis).

Two observations are evident. First, the disparity between the firm's demand if it happens to be the lowest price firm versus not is very large. For example, for $N=2$ the lowest-price demand is up to three times larger than the loyal demand, and for $N=4$ it is up to five times larger ${ }^{7}$. An increase in the number of competitors increases (in relative terms) the additional demand that the firm captures if it is the lowest-price firm. Second, the probability of getting this additional demand decreases in the number of competitors - compare the dashed lines on the left and right subfigures on Figure 2.


Figure 2: The illustration for the inventory risk: the degree of disparity between the firm's demand if it is not lowest price (lower error bars) and the firm's demand if its price is lowest (upper error bars), as a function of $p_{i}$.

[^4]Combining these two factors, i.e., an increase in the relative additional demand with a simultaneous decrease in the probability of getting that additional demand makes the firm's inventory management a lot more difficult as the number of competitors increases.

With $N=2$ competitors, the firm can adopt either a "low" or a "high" price strategy. In the former case, e.g., with $p_{i}=1.5$ (recall that cost $c=1$ ), the firm's expected demand is 245 , which is close to the maximal demand of 262 , and the probability of seeing the maximal demand is $>80 \%$. Hence, the firm can do well by ordering "a lot" of inventory to target its own loyals + bargain-hunters. It will then serve many customers, although at a small margin, and experience the mismatch between its supply and demand only with a small probability. In the latter case, e.g., with $p_{i}=4$, the firm can also do well by ordering "little" inventory to target only its own loyal demand. Indeed, the expected demand in this case is 28 , which is close to the loyals-only demand of 25 , and, because other firms will likely attract the bargain-hunters, the probability that the focal firm will face a shortage is only $3 \%$. That is, with the highprice strategy the firm will serve a small market, but at a high margin, and incur the (small) shortage cost and only with a small probability. The "medium price" strategy, however, is hardly profitable because regardless of how much inventory the firm orders, it will likely incur a large cost of either liquidation or shortage: see how its expected demand line is not close to either of the error bars.

With $N=4$, however, the low-price strategy becomes unprofitable: from Figure 2 the expected demand at $p_{i}=1.5$ is neither low nor high, and the probability of being the lowest price firm is almost $50 \%$. The situation is like the "medium" price strategy with $N=2$. Hence, the firms find the "high price" strategy more appealing. That reduces the exposure to inventory risks and increases the margins to absorb them if they happen.

Further illustrating this point, Figure 3 presents the 3D plots, which depict the densities of the equilibrium price and quantity distributions from the QRE model. The "low" and "high" price strategies clearly reveal themselves: for $N=2$ the peak is at the low prices and high quantities, but as $N$ increases, the peak shifts toward higher prices and lower quantities.


Figure 3: QRE strategy distributions for $N=2,3,4$; in each 3 D plot the right axis is for $q_{i}$ and the left is for $p_{i}$.

Summarizing this discussion, we showed that the counterintuitive price-increase phenomenon that we observed in the Supply Chain game could be driven by the bounded rationality of the managers at the competing firms. Since each of them is unsure about the price that the competitor(s) will charge, as the number of competitors increases, they prefer to err on the side of a higher price and smaller inventory. This hedges from a risk of being undercut of the bargain-hunting demand by a low-price competitor and being "stuck" with the corresponding inventory. Simultaneously, this also hedges from the inverse risk of experiencing large shortages should the competitors charge a high price and the bargain-hunters visit them; per the earlier discussion, even giants like Target are not immune to such possibilities. Increasing the number of competitors increases these risks, and hence the price-increase result follows.

A natural question, unanswered by this result, however, is whether the phenomenon is solely driven by human irrationality. As we discuss next, it is not.

## 5. [Fully Rational] Nash Equilibrium Model

We now consider a situation with fully rational competitors. We present (and solve) two analytical models for price and inventory competition. The first model considers the manufacture-to-stock case we discussed so far, and the second model considers the manufacture-to-order case. Since inventory risk is not present in the latter, comparing these two cases allows us to conclude causality: that is, that the priceincrease phenomenon is observed because of inventory risk, and not just simply with it.

### 5.1 Manufacture-to-Stock (MTS) Case

We follow the model primitives outlined in Section 2 and consider an industry with $N \geq 2$ firms selling identical products and deciding selling prices, $p_{i}$, and a order quantities, $Q_{i}$, before demand is realized. To keep our model parsimonious, we normalize the market size to one and we further assume that $p_{i} \in$ $\left\{p_{L}, p_{H}\right\}$, where, as the labels suggest, $p_{L}<p_{H}$, so that the firm's demand from the loyals is $\frac{\lambda}{N}$ when it charges $p_{L}$, and the demand is $\frac{\lambda-\mu}{N}$ when it charges $p_{H}$, where $\mu \in\left[0, \lambda\left(\frac{p_{H}-p_{L}}{p_{H}-c}\right)\right]$; demand from bargainhunters is $1-\lambda$, as before. The parameter $\mu$ captures the demand elasticity; it is straightforward to express it in terms of the intercept and slope of the linear demand function presented earlier and used in the QRE model. Appendix B. 2 shows that the two-price assumption is not restrictive-under competition with loyal and bargain-hunting customers a continuous-price demand model with $D_{i}\left(p_{i}, \boldsymbol{p}_{-i}\right)$ introduced earlier w.l.o.g collapses to a two-price case in equilibrium.

We now proceed to the analyses of the pricing and quantity equilibrium. Since our main interest is to examine the changes in prices and profits, we focus our attention on the scenario where not all firms set the same price. Let $\bar{N}=\left\lfloor\frac{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}{(1-\lambda)\left(p_{L}-c\right)}\right\rfloor+1, \quad \widehat{N}=\left\lfloor\frac{p_{H}-\lambda p_{L}-\mu\left(p_{H}-c\right)}{(1-\lambda) p_{L}}\right\rfloor+1, \quad \widetilde{N}=$ $\left\lfloor\frac{p_{L}-\lambda p_{H}+\mu\left(p_{H}-c\right)}{c(1-\lambda)}\right\rfloor+1$, and $k^{*}=\left\lfloor\frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}\right\rfloor$, where $\left.\rfloor\right\rfloor$ is the "floor" operator that rounds a number down to the nearest integer; for example, $[3.9]=3$. Lemma 1 presents the firms' equilibrium strategy (The proofs of all results are in Appendix A):

Lemma 1: If $N \geq \max [\bar{N}, \widetilde{N}, \widehat{N}]$ then equilibrium is achieved when $k^{*}$ firms charge the low price ( $p_{L}$ ) and $N-k^{*}$ firms charge the high price $\left(p_{H}\right)$. The corresponding equilibrium order quantities are $\left(\frac{\lambda-\mu}{N}\right)$ for the high-price firms, and $\left(\frac{\lambda}{N}+1-\lambda\right)$ for the low-price firms.

This equilibrium possesses two important properties. First, from the definition of $k^{*}$, the number of firms charging the low price weakly increases with the number of competitors in the industry, $N$.

However, because of the 【 〕 operator, $k^{*}$ may not change when $N$ increases by one. This is because for an additional firm entering the industry, the potential gain from serving the bargain-hunters may not justify the low probability of winning their demand. This additional firm may be better off serving just its loyal customers, for which it would set the high price, order little inventory, and face no inventory risk.

Second, the equilibrium solution would not result in any shortage. This is because a firm that sets the high price sees only loyal demand and sets its order quantity to satisfy it. Similarly, a firm that sets the low price would order enough inventory to serve both types of customers were it to win the bargainhunters' demand. There is clearly some ${ }^{8}$ overstocking in the system as only one low-price firm wins the bargain-hunting demand; overstocking is consistent with Mahajan and van Ryzin (2001).

Having characterized the equilibrium, we proceed to analyzing the average prices. From Lemma 1 the average ${ }^{9}$ price in the market is $E[p]=\frac{\left(N-k^{*}\right)}{N} p_{H}+\frac{k^{*}}{N} p_{L}$, which can be simplified to:

$$
\begin{equation*}
E[p]=p_{H}-\frac{k^{*}}{N}\left(p_{H}-p_{L}\right) \tag{3}
\end{equation*}
$$

Proposition 1 characterizes the impact of the number of firms on the average price; Figure 4 illustrates the result.

Proposition 1: The upper and lower envelopes of the average price under MTS increase in N. Moreover, introducing an arbitrarily small $\varepsilon \downarrow 0$ to account for when $p_{L} / c$ is an integer, $\lim _{\mathrm{N} \rightarrow \infty} \mathrm{k}^{*}=\left\lfloor\frac{p_{L}}{c+\varepsilon}\right\rfloor$, $\lim _{N \rightarrow \infty} E[p]=p_{H}$ and $E[p] \leq \lim _{N \rightarrow \infty} E[p]$.

The main implication of Proposition 1 and Figure 4 is that in the MTS case the average price exhibits an increasing trend. That is, while the average price is not guaranteed ${ }^{10}$ to increase at each $N$, the number of "breakpoints" at which it decreases is finite, while the number of points at which it increases is

[^5]infinite. Technically, recall that in equilibrium $k^{*}$ firms charge the low price, where $k^{*}$ is given by a floor operator, i.e., $k^{*}$ increases by one when its argument crosses an integer. In words, when a firm joins the industry, i.e., $N \rightarrow N+1$, it may choose not to compete for the bargain-hunters (the argument of $k^{*}$ does not cross as integer) because doing so incurs inventory risk that is not worth taking when the market for bargain hunters is already saturated. Therefore, this firm sets the high price to focus on its loyal customers, which involves no inventory risk. In such a case the proportion of the loyal customers that pay the low price decreases from $\frac{k^{*}}{N}$ to $\frac{k^{*}}{N+1}$, which increases the average price.

That said, from Lemma 1, there also exist cases when an increase from $N$ to $N+1$ results in an equilibrium where the additional firm competes for bargain hunters (i.e., the argument of $k^{*}$ crosses an integer); then the average price will decrease. However, per Proposition 1, only a small number of firms would set the low price as $N$ increases. For example, under the parameter settings of Figure $4 \lim _{N \rightarrow \infty} k^{*}=$ $\left\lfloor\frac{p_{L}}{c+\varepsilon}\right\rfloor=3$, i.e., at most three firms would set a low price in the equilibrium. Thus, the price path has two "breakpoints": at $N=7$, when $k^{*}$ changes from 1 to 2 , and at $N=21$, when $k^{*}$ changes from 2 to 3 . Between these breakpoints, as well as for all $N>21$ the average price increases and eventually converges to $p_{H}$; see Figure 4. As stated earlier, cases when the average price increases monotonically also exist.


Figure 4: The average price (dots) and its upper and lower envelopes as a function of $N$ for $p_{H}=11, p_{L}=4, c=$

$$
1, \lambda=0.5, \text { and } \mu=0.01
$$

To summarize, same as in the boundedly rational QRE model, when competing firms decide on the quantity before knowing their demand (MTS), the resultant inventory risk leads to an increasing trend in the average price as the number of competitors grows in the fully rational Nash model as well.

### 5.2 Manufacture-to-Order (MTO) Case

To demonstrate the causal impact of inventory risk, we next consider an MTO case, in which production can be delayed (at no extra cost) until after customers' demand is realized. Such Make-to-order (or Build-to-Order) supply chain strategy revolutionized the manufacturing industry starting with the Just-in-Time Toyota philosophy (Womack et al. 1990) and continuing with its successful implementation by other large corporations such as Dell and BMW (Gunasekaran and Ngai 2005). The growth of additive manufacturing (e.g., using 3D printers to make products at the demand location on an as-needed basis)which some believe is the beginning of the third industrial revolution (The Economist 2012)—also provides ample opportunity for the use of MTO strategies by firms.

The main difference between the MTO and MTS models is that, due to delayed production, there is no inventory risk; the quantity for firm $i$ always equals the realized demand (i.e., $Q_{i}=D$ ), and, thus, the firms only decide on price. Consequently, firm $i$ would have profit $\left(p_{i}-c\right) D$ if it sets price $p_{i}$ and observes/serves demand $D$.

Let $\quad \widehat{N}_{B}=\left\lfloor\left.\frac{\lambda\left(p_{H}-p_{L}\right)}{(1-\lambda)\left(p_{L}-c\right)}+\left(\frac{p_{H}-c}{p_{L}-c}\right)\left(1-\frac{\mu}{1-\lambda}\right) \right\rvert\,+1 \quad\right.$ and $\quad k_{B}^{*}=\left\lfloor\left.\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)} \right\rvert\,\right.$, where the subscript " $B$ " denotes that the thresholds are for the benchmark MTO system. Lemma 2 presents the firms' equilibrium strategy for the make-to-order situation.

Lemma 2: If $N \geq \max \left[\bar{N}, \widehat{N}_{B}\right]$ and $\left(\frac{\lambda}{1-\lambda}\right)-\left(\frac{\mu}{1-\lambda}\right)\left(\frac{p_{H}-c}{p_{H}-p_{L}}\right)>\left(\frac{p_{L}-c}{p_{H}-p_{L}}\right)$ then ${ }^{11}$ the MTO equilibrium is achieved when $k_{B}^{*}$ firms charge the low price $\left(p_{L}\right)$ and ( $N-k_{B}^{*}$ ) firms charge the high price $\left(p_{H}\right)$. The corresponding equilibrium order quantities are $\left(\frac{\lambda-\mu}{N}\right)$ for the high-price firms, $\left(\frac{\lambda}{N}+1-\lambda\right)$ for the firm that charges the low price and wins the bargain customers, and $\frac{\lambda}{N}$ for other low-price firms.

Like the MTS model, we can also define the average price $E\left[p_{B}\right]$, by replacing $k^{*}$ with $k_{B}^{*}$ in equation (3). Then Proposition 2 characterizes the prices under the MTO benchmark.

Proposition 2: The upper and lower envelopes of the average retail price under MTO decrease in $N$.
Moreover, $\lim _{N \rightarrow \infty} k_{B}^{*}=N \frac{\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}, \quad \lim _{N \rightarrow \infty} E\left[p_{B}\right]=p_{H}-\frac{\left(p_{L}-c\right)(1-\lambda)\left(p_{H}-p_{L}\right)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}, E\left[p_{B}\right] \geq \lim _{N \rightarrow \infty} E\left[p_{B}\right]$.

The main implication of Proposition 2 is that in the MTO case the average price exhibits a decreasing trend. Same as in the MTS case, the prices under MTO are not guaranteed to decrease in all cases, however, the main difference is that the number of breakpoints at which the prices decrease is infinite under MTO, as opposed to finite under MTS. Observe that $\lim _{N \rightarrow \infty} k_{B}^{*}=O(N)$ in Proposition 2 and the average price therefore converges to a number strictly less than $p_{H}$.

Summarizing, removing the inventory risk (MTO) leads to the decreasing trend in the average price as the number of competitors grows. Since the MTS and MTO cases are otherwise identical, we therefore established the causal relationship between the increasing prices and inventory risk.

### 5.3 Comparing MTS and MTO Models: Impact of Inventory Risk on Profits

While the main goal of our paper was to understand the impact of inventory risk on prices, the models we developed shed an additional insight on the impact of inventory risk on profits. Recall that in equilibrium

[^6]$k$ firms charge the low price; and $N-k$ firms charge the high price. Under MTS , the total industry profit can be expressed as follows (for more detail see Equations A1-A2 in the proof of Lemma 1):
\[

$$
\begin{equation*}
\Pi=(\lambda-\mu)(E[p]-c)+(1-\lambda)\left(p_{L}-c k^{*}\right)+\frac{k^{*}}{N} \mu\left(p_{L}-c\right) . \tag{4}
\end{equation*}
$$

\]

The first and third terms in (4) correspond to all firms' guaranteed profit from selling to their loyal customers, and the second term of $\Pi$ is the industry profit from selling to the bargain-hunters. The multiplier $k^{*}$ in the second term of $\Pi$ reflects that $k^{*}$ firms that stock the inventory to serve bargain hunters, but only one firm that serves the realized demand (see Lemma 1).

Under the MTO benchmark, the industry profit is given by:

$$
\begin{equation*}
\Pi_{\mathrm{B}}=(\lambda-\mu)\left(E\left[p_{B}\right]-c\right)+(1-\lambda)\left(p_{L}-c\right)+\frac{k_{B}^{*}}{N} \mu\left(p_{L}-c\right) \tag{5}
\end{equation*}
$$

The industry profit under MTO in (5) differs from that under MTS in (4), reflecting the absence of inventory risk. Specifically, there is no multiplier of $k_{B}^{*}$ in the second term of $\Pi_{\mathrm{B}}$ as only one low-price firm (one that "wins" the bargain-hunters' demand) orders inventory to serve them.

Proposition 3 presents the comparison between these two cases, and Figure 5 overlays the equilibrium prices (left panel) and industry profits (right panel) for the MTS case with those of the MTO case. To facilitate the comparison, we use the same parameter settings as in Figure 4.

Proposition 3: $k^{*} \leq k_{B}^{*}, E[p] \geq E\left[p_{B}\right]$ and $\lim _{N \rightarrow \infty} \Pi \geq \lim _{N \rightarrow \infty} \Pi_{B}$.
The main implication of Proposition 3 is that the industry profit (and, consequently, the average profit per firm) is higher ${ }^{12}$ under MTS than under the MTO. This result may initially seem surprising as under MTS the firm runs an inflexible (and inefficient) operation where inventory is ordered based on a demand forecast that may be incorrect. How can this be better than running a flexible operation where the products can be made (at the same cost) after the demand is realized? Per Proposition 3 this is because, in the presence of inventory risk, fewer firms charge the low price, $k^{*} \leq k_{B}^{*}$ : the consequences of not winning the bargain-hunters' demand are more severe in the MTS case and thus fewer firms compete for

[^7]them. Equivalently, competition is less intense in the presence of inventory risk and firms naturally prefer that. It can be shown that the profit gain from inventory risk equals $\lim _{N \rightarrow \infty}\left(\Pi-\Pi_{B}\right)=(1-\lambda)\left(p_{L}-\right.$ $\left\lfloor\left.\frac{p_{L}}{c+\varepsilon} \right\rvert\, c\right)$, i.e., it increases in the fraction of bargain-hunters, and, consequently in the amount of inventory risk in the market.


Figure 5: Impact of the number of firms ( $N$ ) to the average price (left) and industry profit industry profit (right) for $p_{H}=11, p_{L}=4, c=1, \lambda=0.5$, and $\mu=0.01$. Solid dots denote: MTS, crosses denote: MTO

Summarizing, the counter-intuitive increase in market prices due to inventory risk has an equally counter-intuitive implication for profits: inventory risk softens competition and firms may therefore prefer to incur rather than avoid it.

## 6. Conclusions

The goal of this paper is to understand the impact of inventory risk on market prices, and, in particular, to explain a surprising effect that we observed from playing a competitive extension of the Beer Game with graduate students and executives. In contradiction to common intuition we observed that an increase in the number of competing retailers could lead to a general increase in the equilibrium market price in the situation where firms compete on both price and quantity/inventory/capacity.

We used multiple methods, and considered both behavioral, boundedly-rational motives (captured via the QRE model) and fully-rational motives (captured via the Nash Equilibrium model). Both methods agree in qualitative findings: the increasing trend in prices is the result of inventory risk - the potential mismatch between the firm's inventory and demand, that occurs if the firm incorrectly judges how much additional demand it will generate if it lowers the price. Inventory risk increases with the number of competitors, making firms' inventory management more difficult and costly, and triggering an increase in the firms’ desire to charge higher prices and avoid competing for the low-price demand. This in turn softens competition, and the benefits from that can be so large that firms could prefer operating in situations with rather than without inventory risk. This, of course, does not mean that each individual firm would not be interested in improving its operations, but rather that the situation is of the prisoner's dilemma type: all firms would be better off if they collude and operate with inventory risk but with less intense competition.

An important high-level contribution of our paper is in recognizing that the inventory risk that firms incur in many practical settings (as they build inventories/capacities in advance of knowing demand), has the potential to change the direction of how researchers and practitioners think about competitive outcomes. Business executives, who contemplate entering a market with a low-price strategy-such as those at Target-need to determine whether doing so is worth the additional inventory risks. They also ought to consider whether improving supply-chain responsiveness to combat those risks would lead to a long-term sustainable competitive advantage or provoke other firms and eventually erode profits for all. Government officials also need to realize that encouraging higher-cost, fast response supply chains would not only reduce inventory inefficiencies but could also lower prices. In contrast, efforts to increase competition in situations with low supply chain responsiveness would increase prices and therefore hurt consumers.

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## Appendix A: Proofs

## Proof of Lemma 1:

To prove Lemma 1 we first derive the order quantities for a firm that charges the high price and for one that charges the low price. We then show that there are $N-k^{*}$ high-price firms and $k^{*}$ low-price firms in the equilibrium. Finally, we show that the conditions on $N$ in the statement of the Lemma ensure $k^{*} \in$ $[1, N-1]$. Note that the range of the order quantity is between $\frac{\lambda-\mu}{N}$ and $\frac{\lambda-\mu}{N}+1-\lambda$ when a firm charges the high price and is between $\frac{\lambda}{N}$ and $\frac{\lambda}{N}+1-\lambda$ when a firm charges the low price. This range means that the firms would order at least the demand from their loyal customers and at most the demand from both types of customers. Ordering outside of this range is never optimal.

A firm that charges the high price only serves its loyal customers, so its order quantity is $\frac{\lambda-\mu}{N}$, and its profit is

$$
\begin{equation*}
\left(p_{H}-c\right) \frac{\lambda-\mu}{N} \tag{A1}
\end{equation*}
$$

A firm that charges the low price has the profit of $\left(p_{L}-c\right) Q-s\left(\frac{\lambda}{N}+1-\lambda-Q\right)$ if it wins the bargain-hunters' demand, and $p_{L} \frac{\lambda}{N}-c Q$ otherwise. The average profit of such a firm is $\left(\frac{p_{L}+s}{k}-c\right) Q+$ $\left(1-\frac{1}{k}\right) p_{L} \frac{\lambda}{N}-\frac{s}{k}\left(\frac{\lambda}{N}+1-\lambda\right)$. The definition of $k^{*}$ guarantees that $\frac{p_{L}+s}{k^{*}} \geq c$, because $k^{*} \leq \frac{p_{L}+s}{c} \Leftarrow$ $\frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)} \leq \frac{p_{L}+s}{c} \Leftrightarrow \lambda\left(p_{L}+s\right)\left(p_{H}-p_{L}\right)+s c N(1-\lambda) \geq \mu\left(p_{L}+s\right)\left(p_{H}-c\right)$, which is true for $\mu \in\left[0, \frac{\lambda\left(p_{H}-p_{L}\right)}{p_{H}-c}\right]$. Therefore, the low-price firm would set the order quantity of $\frac{\lambda}{N}+1-\lambda$, and the average profit is

$$
\begin{equation*}
\left(p_{L}-c\right) \frac{\lambda}{N}+\left(\frac{p_{L}}{k}-c\right)(1-\lambda) \tag{A2}
\end{equation*}
$$

A firm that charges the low price will not deviate if and only if $\left(p_{L}-c\right) \frac{\lambda}{N}+\left(\frac{p_{L}}{k^{*}}-c\right)(1-\lambda) \geq$ $\left(p_{H}-c\right) \frac{\lambda-\mu}{N} \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}} \leq \frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}}-c\right)$, and a firm that charges the high price will not
deviate if and only if $\quad\left(p_{H}-c\right) \frac{\lambda-\mu}{N}>\left(p_{L}-c\right) \frac{\lambda}{N}+\left(\frac{p_{L}}{k^{*}+1}-c\right)(1-\lambda) \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}}>$ $\frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}+1}-c\right)$. As such, the necessary and sufficient condition for equilibrium is:

$$
\begin{equation*}
\frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}+1}-c\right)<\frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H^{-}}-c}{p_{H}-p_{L}} \leq \frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}}-c\right) \tag{A3}
\end{equation*}
$$

Equation A3 can be expressed as $\frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}-1<k^{*} \leq \frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}$, which is the definition of $k^{*}$. In other words, the alternative equilibrium where $\hat{k}$ firms charge the low pricefor $\hat{k} \neq k^{*}$ and $\hat{k} \geq 1$ "-does not exist because it contradicts the Equation A3 and the definition of $k^{*}$. However, since by definition $k^{*} \in[1, N-1]$, the above holds under the following conditions:

$$
\begin{aligned}
& -\quad k^{*} \geq 1 \Leftrightarrow \frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)} \geq 1 \Leftrightarrow N \geq \frac{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}{(1-\lambda)\left(p_{L}-c\right)} \Leftrightarrow N \geq \bar{N} \\
& -\quad k^{*} \leq N-1 \Leftrightarrow \frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}<N \Leftrightarrow N>\frac{p_{L}-\lambda p_{H}+\mu\left(p_{H}-c\right)}{c(1-\lambda)} \Leftrightarrow N \geq \widetilde{N} .
\end{aligned}
$$

We finally need to check that $k^{*}=0$ is not an equilibrium. If all firms charge the high price and order some quantity $Q$, then the average profit would be $\left(\frac{p_{H}+s}{N}-c\right) Q+\left(1-\frac{1}{N}\right) p_{H} \frac{\lambda-\mu}{N}-\frac{s}{N}\left(\frac{\lambda-\mu}{N}+1-\right.$ $\lambda$ ). When $\frac{p_{H}+s}{N} \geq c$, the profit would be increasing in $Q$ and the firm would therefore set the highest order quantity possible, $\frac{\lambda-\mu}{N}+1-\lambda$, for the profit of $\left(p_{H}-c\right) \frac{\lambda-\mu}{N}+\left(\frac{p_{H}}{N}-c\right)(1-\lambda)$. A firm that deviates would set the order quantity $\frac{\lambda}{N}+1-\lambda$, resulting in profit of $\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right)$. This is not an equilibrium if and only if $\quad\left(p_{H}-c\right) \frac{\lambda-\mu}{N}+\left(\frac{p_{H}}{N}-c\right)(1-\lambda) \leq\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right) \Leftrightarrow \frac{\lambda}{1-\lambda}-$ $\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}} \leq \frac{N}{p_{H}-p_{L}}\left(p_{L}-\frac{p_{H}}{N}\right) \Leftrightarrow N \geq \widehat{N}$. On the other hand, when $\frac{p_{H}+s}{N}<c$, the firm would set the smallest order quantity possible, $\frac{\lambda-\mu}{N}$, for the profit of $\left(p_{H}-c\right) \frac{\lambda-\mu}{N}-\frac{s}{N}(1-\lambda)$. This is not an equilibrium if and only if $\left(p_{H}-c\right) \frac{\lambda-\mu}{N}-\frac{s}{N}(1-\lambda)<\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right) \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}}<$ $\frac{N}{p_{H}-p_{L}}\left(p_{L}-c+\frac{s}{N}\right)$, which is true because $\frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}}-c\right)<\frac{N}{p_{H}-p_{L}}\left(p_{L}-c+\frac{s}{N}\right)$.

## Proof of Proposition 1:

Recall that $k^{*}=\left\lfloor\frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}\right\rfloor$ and $E[p]$ is the average price when $k=k^{*}$. Define $\overline{\bar{k}}^{*}=$ $\frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}$, i.e., it equals to $k^{*}$ but without the floor operator. Moreover, define $\overline{\overline{E[p]}}$ as the average price when $k=\overline{\bar{k}}^{*}$, i.e., assume that firms are infinitely divisible. Recall that $k$ is the number of firms charging low price. Because $\overline{\bar{k}}^{*} \geq k^{*}$, there are more (fractional) firms charging low price if $k=\overline{\bar{k}}^{*}$ than if $k=k^{*}$, and therefore $\overline{\overline{E[p]}}$ is the lower envelope of $E[p]$. Similarly, $\overline{\bar{k}}^{*}-1 \leq k^{*}$, there are fewer, in the weak sense, (fractional) firms charging low price if $k=\overline{\bar{k}}^{*}-1$ than if $k=k^{*}$. Thus, an $\overline{\overline{E[p]}}$ constructed with $k=\overline{\bar{k}}^{*}-1$ is an upper envelope of $E[p]$. It is straightforward to show that $\overline{\bar{k}}^{*}$ increases in $N$ and thus so do both upper and lower envelopes. Lastly, it is easy to observe that $\lim _{N \rightarrow \infty} k^{*}=\left\lfloor\frac{p_{L}}{c+\varepsilon}\right\rfloor$, where we introduce $\varepsilon \downarrow 0$ to account for the case when $p_{L} / c$ is an integer. Then, $\lim _{N \rightarrow \infty} E[p]=p_{H}$ because $\lim _{N \rightarrow \infty} \frac{k^{*}}{N}=0$, and therefore $\lim _{N \rightarrow \infty} E[p] \geq E[p]$.

## Proof of Lemma 2:

We use the same logic here as for the proof of Lemma 1. In particular, a firm that charges the high price only serves its loyal customers, so it orders $\frac{\lambda-\mu}{N}$ for the profit of $\left(p_{H}-c\right) \frac{\lambda-\mu}{N}$. However, a firm that charges the low price would wait until it learns if it won the bargain hunters' demand, in which case it would order $\frac{\lambda}{N}+1-\lambda$; it would otherwise order $\frac{\lambda}{N}$. The average profit of such a firm would be ( $p_{L}-$ c) $\left(\frac{\lambda}{N}+\frac{1-\lambda}{k}\right)$.

A firm that charges the low price would not deviate if and only if $\left(p_{L}-c\right)\left(\frac{\lambda}{N}+\frac{1-\lambda}{k_{B}^{*}}\right) \geq$ $\left(p_{H}-c\right) \frac{\lambda-\mu}{N} \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}} \leq \frac{N}{p_{H}-p_{L}} \frac{p_{L}-c}{k_{B}^{*}}$, and a firm that charges the high price would not deviate if and only if $\left(p_{H}-c\right) \frac{\lambda-\mu}{N}>\left(p_{L}-c\right)\left(\frac{\lambda}{N}+\frac{1-\lambda}{k_{B}^{*}+1}\right) \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}}>\frac{N}{p_{H}-p_{L}} \frac{p_{L}-c}{k_{B}^{*}+1}$. So, the necessary and sufficient condition for the equilibrium is:

$$
\begin{equation*}
\frac{N}{p_{H}-p_{L}} \frac{p_{L}-c}{k_{B}^{*}+1}<\frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}} \leq \frac{N}{p_{H}-p_{L}} \frac{p_{L}-c}{k_{B}^{*}} \tag{A4}
\end{equation*}
$$

Equation A4 can be expressed as $\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}-1<k_{B}^{*} \leq \frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}$, which is the definition of $k_{B}^{*}$. The alternative equilibrium, where $\hat{k}_{B} \neq k_{B}^{*}$ and $\hat{k}_{B} \geq 1$ firms charge low price, does not exist because it contradicts Equation A4 and the definition of $k_{B}^{*}$.

Since by definition, $k_{B}^{*} \in[1, N-1]$, the above holds under the following conditions:

$$
\begin{aligned}
& -\quad k_{B}^{*} \geq 1 \Leftrightarrow \frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)} \geq 1 \Leftrightarrow N \geq \frac{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}{(1-\lambda)\left(p_{L}-c\right)} \Leftrightarrow N \geq \bar{N} ; \\
& -\quad k_{B}^{*} \leq N-1 \Leftrightarrow\left|\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}\right| \leq N-1 \Leftrightarrow \frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}<N \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}}> \\
& \\
& p_{H}-p_{L} .
\end{aligned}
$$

We finally need to check that $k_{B}^{*}=0$ is not an equilibrium. If all firms charge the high price, the average profit would be $\left(p_{H}-c\right)\left(\frac{\lambda-\mu}{N}+\frac{1-\lambda}{N}\right)$. A firm that deviates would set order quantity $\frac{\lambda}{N}+1-\lambda$, resulting in profit $\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right)$. This is not an equilibrium if and only if $\left(p_{H}-c\right)\left(\frac{\lambda-\mu}{N}+\frac{1-\lambda}{N}\right) \leq$ $\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right) \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}} \leq \frac{N}{p_{H}-p_{L}}\left(p_{L}-\frac{p_{H}}{N}-\frac{N-1}{N} c\right) \Leftrightarrow N \geq \widehat{N}_{B}$.

## Proof of Proposition 2:

We use a similar logic as the proof of Proposition 1 to show this proposition. Specifically, recall that $k_{B}^{*}=\left\lfloor\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}\right\rfloor$ and $E[p]$ is the average price when $k=k_{B}^{*}$. Define $\overline{\bar{k}}_{B}^{*}=\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}$, i.e., $k_{B}^{*}$ but without the integer operator. Moreover, define $\overline{\bar{E} \overline{[p]_{L}}}$ as the average price when using $k=\overline{\bar{k}}_{B}^{*}-1$. There are fewer (fractional) firms charging low price if $k=\overline{\bar{k}}_{B}^{*}-1$ than if $k=k_{B}^{*}$ because $\overline{\bar{k}}_{B}^{*}-1$ is the lower envelope of $k_{B}^{*}$. Hence, $\overline{\overline{E[p]_{L}}}$ is the upper envelope of $E[p]$. It is straightforward to show that $\overline{\overline{E[p]_{L}}}$ decreases in $N$. Hence, the general trend of $E[p]$ will be decreasing in $N$. Lastly, by definition of Equation 3, the average price $E\left[p_{B}\right]$ is always larger when there is an $\left\rfloor\right.$ operator in $k_{B}^{*}$ than when there isn't. Note that $\lim _{N \rightarrow \infty}\left|\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}\right|=\lim _{N \rightarrow \infty} \frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}=\infty$ (i.e., the $\left\rfloor\right.$ operator in $k_{B}^{*}$ is not
necessary in this case when $N$ is large), so $\lim _{N \rightarrow \infty} \frac{k_{B}^{*}}{N}=\frac{\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}$ and $\lim _{N \rightarrow \infty} E\left[p_{B}\right]=p_{H}-$ $\frac{\left(p_{L}-c\right)(1-\lambda)\left(p_{H}-p_{L}\right)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}$. Hence, we obtain that $E\left[p_{B}\right] \geq \lim _{N \rightarrow \infty} E\left[p_{B}\right]$.

Proof of Proposition 3:
$\left.k^{*} \leq k_{B}^{*} \Leftrightarrow \left\lvert\, \frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}\right.\right\rfloor \leq\left\lfloor\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)} \left\lvert\, \Leftrightarrow \frac{p_{L} N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N(1-\lambda)-\mu\left(p_{H}-c\right)}<\right.\right.$
$\frac{N\left(p_{L}-c\right)(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)} \Leftrightarrow N>\frac{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}{(1-\lambda)\left(p_{L}-c\right)}$, which is true because $N \geq \bar{N} . E[p] \geq E\left[p_{B}\right] \Leftrightarrow p_{H}-$ $\frac{k^{*}}{N}\left(p_{H}-p_{L}\right) \geq p_{H}-\frac{k_{B}^{*}}{N}\left(p_{H}-p_{L}\right) \Leftrightarrow k^{*} \leq k_{B}^{*}$, which is true. Lastly, $\lim _{N \rightarrow \infty} \Pi=\lambda\left(\lim _{N \rightarrow \infty} E[p]-c\right)+$ $(1-\lambda)\left(p_{L}-c \lim _{N \rightarrow \infty} k^{*}\right)-\left(1-\frac{\lim _{N \rightarrow \infty} k^{*}}{N}\right) \mu\left(p_{H}-c\right)=(\lambda-\mu)\left(p_{H}-c\right)+(1-\lambda)\left(p_{L}-\left\lfloor\left.\frac{p_{L}}{c+\varepsilon} \right\rvert\, c\right)\right.$.
$\lim _{N \rightarrow \infty} \Pi_{B}=\lambda\left(\lim _{N \rightarrow \infty} E\left[p_{B}\right]-c\right)+(1-\lambda)\left(p_{L}-c\right)-\left(1-\lim _{N \rightarrow \infty} \frac{k_{B}^{*}}{N}\right) \mu\left(p_{H}-c\right)=(\lambda-\mu)\left(p_{H}-c\right)$.
$\lim _{N \rightarrow \infty} \Pi \geq \lim _{N \rightarrow \infty} \Pi_{\mathrm{B}} \Leftrightarrow(\lambda-\mu)\left(p_{H}-c\right)+(1-\lambda)\left(p_{L}-\left\lfloor\left.\frac{p_{L}}{c+\varepsilon} \right\rvert\, c\right) \geq(\lambda-\mu)\left(p_{H}-c\right)\right.$, which is true because $p_{L} \geq\left\lfloor\frac{p_{L}}{c+\varepsilon}\right\rfloor c$.

## Appendix B: Robustness of results

In this appendix, we examine the robustness of our results with respect to the key modeling assumptions.

## Appendix B.1: "Winner-takes-all" vs fractional allocation of bargain-hunting demand

This analyses below consider the Nash equilibrium case. For the QRE we reran our simulation with the logit demand function so that the size of the bargain hunters' demand decreased in the difference from the lowest price firm and still observed the price-increase phenomenon; details available upon request.

So far we assumed that whenever $k \geq 2$ firms have the same low(est) price, the bargain-hunters all visit the same firm, selected at random among the lowest-price firms with probability $1 / k$. The thresholds $\bar{N}=\left\lfloor\frac{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}{(1-\lambda)\left(p_{L}-c\right)}\right\rfloor+1, \widehat{N}=\left\lfloor\frac{p_{H}-\lambda p_{L}-\mu\left(p_{H}-c\right)}{(1-\lambda) p_{L}}\right\rfloor+1$ and $\widetilde{N}=\left\lfloor\frac{p_{L}-\lambda p_{H}+\mu\left(p_{H}-c\right)}{c(1-\lambda)}\right\rfloor+1$ were used to provide bounds necessary for the existence of the equilibrium solution (cf. Lemma 1). In this appendix, we extend this approach to a more general setting where each of the $k$ low-price firms receives a portion of the bargain-hunters' demand. In particular, suppose that a fraction $\beta$ of the bargain-hunters visit one firm, and the remaining fraction $(1-\beta)$ equally spreads to all low-price firms. Therefore, all low-priced firms would not only have $\frac{\lambda}{N}$ loyal customers, but would also be guaranteed to have $\frac{(1-\beta)(1-\lambda)}{k}$ bargain-hunters, and one of these firms would have an addition $\beta(1-\lambda)$ bargain-hunters.

Before we proceed with proving this extended version of Lemma 1, referred to as Lemma A1 below, we make the following critical remark: empirical evidence suggests, and our proof requires $\beta$ to be relatively large. For evidence, we refer the reader to a Science Magazine article by Salganik et al. (2006) [which has over 1500 citations as of summer 2018]. Salganik et al. used field experiments to understand how word-of-mouth and social influence shape demand for products of otherwise identical quality. They observed an unambiguous pattern: one random product emerged as a "leader" and attracted the vast majority of demand, of which the others received only a minor fraction; they argue that the nature of
human interactions and search coupled with listing algorithms used by search engines ensure this pattern. We believe the same would hold for the bargain-hunters’ demand as well: once word-of-mouth spreads that some "firm X" has the lowest price, the majority of bargain-hunters would favor that firm, with only a few patronizing others. Formally, we assume $\exists \bar{\beta}$ s.t. $\beta \in[\bar{\beta}, 1]$, which is the condition required for the equilibrium result in Lemma A1 below to hold. Intuitively, this assumption is needed because if $\beta$ is small (in the limit think of $\beta=0$ ), all firms would receive a nearly identical fraction of the total demand and, hence, would face no inventory risk; our "story" clearly would not apply in that case.

Lemma A1, below, extends the equilibrium presented in Lemma 1 (under the winner-takes-all allocation of bargain-hunters' demand) to the $\beta$-allocation described above. Since the structure of the equilibrium in the two Lemmas is the same (apart from the $N$-thresholds and $k^{*}$, naturally, becoming functions of $\beta$ ), and all other results in the paper rely on this structure, all results presented in the paper hold under this more general model of $\beta$-allocation. It is straightforward to verify that with $\beta \rightarrow 1$ Lemma A1 below converges to Lemma 1 in the main body of the paper.

Lemma A1: Define $\bar{N}(\beta)=\left\lfloor\frac{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}{(1-\lambda)\left(p_{L}-c\right)}\right\rfloor+1, \widehat{N}(\beta)=\left\lfloor\frac{p_{H}-\lambda p_{L}-\mu\left(p_{H}-c\right)-c(1-\beta)(1-\lambda)}{(1-\lambda)\left(p_{L}-c(1-\beta)\right)}\right\rfloor+1$ and $\widetilde{N}(\beta)=\left\lfloor\frac{p_{L}-\lambda p_{H}+\mu\left(p_{H}-c\right)-c(1-\beta)(1-\lambda)}{\beta c(1-\lambda)}\right\rfloor+1 \quad$ and $\quad k^{*}(\beta)=\left\lfloor\frac{\left(p_{L}-c(1-\beta)\right) N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N \beta(1-\lambda)-\mu\left(p_{H}-c\right)}\right\rfloor$. Consider a situation where a fraction $\beta$ of the bargain-hunters visit one firm, and the remaining fraction ( $1-\beta$ ) equally spreads to all other low-price firms. If $N \geq \max [\bar{N}(\beta), \widetilde{N}(\beta), \widehat{N}(\beta)]$ then equilibrium is achieved when $k^{*}(\beta)$ firms charge the low price $\left(p_{L}\right)$ and $N-k^{*}(\beta)$ firms charge the high price $\left(p_{H}\right)$. The corresponding equilibrium order quantities are $\left(\frac{\lambda-\mu}{N}\right)$ for the high-price firms, and $\left(\frac{\lambda}{N}+1-\lambda\right)$ for the low-price firms.

## Proof of Lemma A1:

A firm that charges the high price only serves its loyal customers, so its profit is given in (A1). A firm that charges the low price has the profit of $\left(p_{L}-c\right) Q-s\left(\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k}+\beta(1-\lambda)-Q\right)$ if it wins the
bargain-hunters' demand, and $p_{L}\left(\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k}\right)-c Q$ otherwise. The average profit of such a firm is $\left(\frac{p_{L}+s}{k}-c\right) Q+\left(1-\frac{1}{k}\right) p_{L}\left(\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k}\right)-\frac{s}{k}\left(\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k}+\beta(1-\lambda)\right)$. The definition of $k^{*}(\beta)$ guarantees that $\frac{p_{L}+s}{k^{*}(\beta)} \geq c$, because $k^{*}(\beta) \leq \frac{p_{L}+s}{c} \Leftarrow \frac{\left(p_{L}-c(1-\beta)\right) N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N \beta(1-\lambda)-\mu\left(p_{H}-c\right)} \equiv \bar{\beta}$, which can be shown to be true. Therefore, the low-price firm would set the order quantity of $\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k}+\beta(1-\lambda)$, and the average profit is $\left(p_{L}-c\right)\left(\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k}\right)+\left(\frac{p_{L}}{k}-c\right)(1-\lambda) \beta$.

A firm that charges the low price would not deviate if and only if $\left(p_{L}-c\right)\left(\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k^{*}(\beta)}\right)+$ $\left(\frac{p_{L}}{k^{*}(\beta)}-c\right)(1-\lambda) \beta \geq\left(p_{H}-c\right) \frac{\lambda-\mu}{N} \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}} \leq \frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}(\beta)}-\frac{c\left(1-\beta+k^{*} \beta\right)}{k^{*}(\beta)}\right)$, and a firm that charge the high price would not deviate if and only if $\left(p_{H}-c\right) \frac{\lambda-\mu}{N}>\left(p_{L}-c\right)\left(\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k^{*}(\beta)+1}\right)+$ $\left(\frac{p_{L}}{k^{*}(\beta)+1}-c\right)(1-\lambda) \beta \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}}>\frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}(\beta)+1}-\frac{c\left(1-\beta+\left(k^{*}(\beta)+1\right) \beta\right)}{k^{*}(\beta)+1}\right)$. So the necessary and sufficient condition for equilibrium is $\frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}(\beta)+1}-\frac{c\left(1-\beta+\left(k^{*}(\beta)+1\right) \beta\right)}{k^{*}(\beta)+1}\right)<\frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}} \leq$ $\frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}(\beta)}-\frac{c\left(1-\beta+\beta k^{*}(\beta)\right)}{k^{*}(\beta)}\right)$. Since by definition $k^{*}(\beta) \in[1, N-1]$, the above holds under the following conditions:

$$
\begin{aligned}
& -\quad k^{*}(\beta) \geq 1 \Leftrightarrow \frac{\left(p_{L}-c(1-\beta)\right) N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N \beta(1-\lambda)-\mu\left(p_{H}-c\right)} \geq 1 \Leftrightarrow N \geq \frac{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}{(1-\lambda)\left(p_{L}-c\right)} \Leftrightarrow N \geq \bar{N}(\beta) ; \\
& -\quad k^{*}(\beta) \leq N-1 \Leftrightarrow \frac{\left(p_{L}-c(1-\beta)\right) N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)+c N \beta(1-\lambda)-\mu\left(p_{H}-c\right)}<N \Leftrightarrow N>\frac{p_{L}-\lambda p_{H}+\mu\left(p_{H}-c\right)-c(1-\beta)(1-\lambda)}{\beta c(1-\lambda)} \Leftrightarrow \\
& \\
& N \geq \widetilde{N}(\beta) .
\end{aligned}
$$

We finally need to check that $k^{*}(\beta)=0$ is not an equilibrium. If all firms charge the high price and order some quantity $Q$, then the average profit would be $\left(\frac{p_{H}+s}{N}-c\right) Q+\left(1-\frac{1}{N}\right) p_{H}\left(\frac{\lambda-\mu}{N}+\right.$ $\left.\frac{(1-\beta)(1-\lambda)}{N}\right)-\frac{s}{N}\left(\frac{\lambda-\mu}{N}+\frac{(1-\beta)(1-\lambda)}{N}+(1-\lambda) \beta\right)$. When $\frac{p_{H}+s}{N} \geq c$, the profit would be increasing in $Q$ and the firm would therefore set the highest order quantity possible, $\frac{\lambda-\mu}{N}+\frac{(1-\beta)(1-\lambda)}{N}+(1-\lambda) \beta$, for the
profit of $\left(p_{H}-c\right)\left(\frac{\lambda-\mu}{N}+\frac{(1-\beta)(1-\lambda)}{N}\right)+\left(\frac{p_{H}}{N}-c\right)(1-\lambda) \beta$. A firm that deviates would set the order quantity $\frac{\lambda-\mu}{N}+\frac{(1-\beta)(1-\lambda)}{N}+(1-\lambda) \beta$, resulting in a profit of $\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right)$. This is not an equilibrium if and only if $\left(p_{H}-c\right)\left(\frac{\lambda-\mu}{N}+\frac{(1-\beta)(1-\lambda)}{N}\right)+\left(\frac{p_{H}}{N}-c\right) \beta(1-\lambda) \leq\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right) \Leftrightarrow$ $N \geq \widehat{N}(\beta)$. When $\frac{p_{H}+s}{N}<c$, the firm would set $Q=\frac{\lambda-\mu}{N}+\frac{(1-\beta)(1-\lambda)}{N}$, for the profit of $\left(p_{H}-c\right)\left(\frac{\lambda-\mu}{N}+\right.$ $\left.\frac{(1-\beta)(1-\lambda)}{N}\right)-\frac{s}{N} \beta(1-\lambda)$. This is not an equilibrium if and only if $\left(p_{H}-c\right)\left(\frac{\lambda-\mu}{N}+\frac{(1-\beta)(1-\lambda)}{N}\right)-$ $\frac{s}{N} \beta(1-\lambda)<\left(p_{L}-c\right)\left(\frac{\lambda}{N}+1-\lambda\right) \Leftrightarrow \frac{\lambda}{1-\lambda}-\frac{\mu}{1-\lambda} \frac{p_{H}-c}{p_{H}-p_{L}}<\frac{N}{p_{H}-p_{L}}\left(p_{L}-c+\frac{s}{N} \beta-\frac{\left(p_{H}-c\right)(1-\beta)}{N}\right)$. This is true because $\frac{N}{p_{H}-p_{L}}\left(\frac{p_{L}}{k^{*}(\beta)}-\frac{c\left(1-\beta+k^{*} \beta\right)}{k^{*}(\beta)}\right)<\frac{N}{p_{H}-p_{L}}\left(p_{L}-c+\frac{s}{N} \beta-\frac{\left(p_{H}-c\right)(1-\beta)}{N}\right)$ and the largest value of the left-hand side is when $k^{*}(\beta)=1$.

We close this appendix by noting that it is not hard to "flip" the above lemma to the case of $\beta \in$ $[0, \bar{\beta}]$ and show that the equilibrium will have $k_{1}^{*}(\beta)=\frac{\left(\left(p_{L}-c\right)(1-\beta)-s \beta\right) N(1-\lambda)}{\lambda\left(p_{H}-p_{L}\right)-\mu\left(p_{H}-c\right)}$ charging $p_{L}$ and ordering $\frac{\lambda}{N}+\frac{(1-\beta)(1-\lambda)}{k}$. Such an equilibrium does not result in overstocking; rather, the firm that wins the bargain -hunting demand experience a stock-out of $\beta(1-\lambda)$. Further, the number of firms charging the low price scales linearly with N ; hence, the price-increasing trend of Proposition 1c no longer exists. This qualitative reversal of behavior is natural since the firms face little inventory risk when $\beta$ is small.

## Appendix B.2: H/L vs continuous prices

In this appendix, we show that, given the fundamental setup of our model (symmetric firms selling to a mixture of loyal and bargain-hunting customers), only two possible prices, i.e., $p_{i} \in\left\{p_{L}, p_{H}\right\}$, emerge in equilibrium even when firms are free to choose from a continuum of prices. Specifically, when $a=$ $\frac{2 p_{L}(\lambda-\mu)-c \lambda}{N\left(p_{L}-c\right)}, b=\frac{\lambda-2 \mu}{N\left(p_{L}-c\right)}$, and $p_{H}=p_{L}+\frac{\mu\left(p_{L}-c\right)}{\lambda-2 \mu}$. Then, the two-price model in Section 5 becomes the continuous price model presented in Section 3.

Since firms are symmetric, it is natural that there exists a lower-bound on the price, which w.l.o.g. we denote as $p_{L}$. Firms in practice have fixed costs (overhead, sales, administrative expenses, etc.), hence $p_{L}>c$. By definition, loyal customers decide to purchase based only on the price at their firm of choice. Thus, if firm $i$ only targets its loyal customers, it would set a price that maximizes $\left(p_{i}-c\right) D^{L}\left(p_{i}\right)$. It can easily be shown that there exists a unique optimal price, which we call $p_{H}$ for notation consistency, that satisfies the first order condition $\frac{\partial D^{L}\left(p_{H}\right)}{\partial p_{H}}\left(p_{H}-c\right)+D^{L}\left(p_{H}\right)=0$. While we purposefully keep the $D^{L}$ function general, it is naturally also restricted so that the resultant $p_{H}>p_{L}$.

Let $\hat{p}=\min \left[\boldsymbol{p}_{-i}\right]$. By definition, bargain-hunters all switch to the firm with the lowest price, which implies three properties of $D^{B}$ :

1. First, if a firm charges a price that is strictly higher than the lowest price of other firms, it will receive a zero demand: i.e., $D^{B}\left(\hat{p}+\varepsilon, \boldsymbol{p}_{-i}\right)=0, \forall \varepsilon$;
2. Second, if a firm charges a price that is strictly lower than the lowest price of other firms, then it will receive a much larger demand than it would receive otherwise: i.e., $D^{B}\left(\hat{p}-\varepsilon, \boldsymbol{p}_{-i}\right) \gg D^{B}\left(\hat{p}, \boldsymbol{p}_{-i}\right), \forall \varepsilon ;$
3. Finally, if the lowest price is charged by several firms, then each such firm has a non-zero expected demand: $E\left[D^{B}\left(\hat{p}, \boldsymbol{p}_{-i}\right)\right]>0$

Combining properties [1] and [3], it is obvious that if firm $i$ is one of $\{k\}$ firms that compete for bargain-hunting customers, then all such $k$ firms will charge the same price, call it $\bar{p}$, and for notational convenience. Let the bold $\overline{\boldsymbol{p}}$ refer to a $k$-vector each element of which equals $\bar{p}$. Suppose $\bar{p}>p_{L}$ and let $q^{*}(\bar{p}, \overline{\boldsymbol{p}})$ be the optimal order quantity for firm $i$ given that all firms price at $\bar{p}$. The firm's expected profit from charging $\bar{p}$ then equals: $\bar{p} E\left[\min \left[q^{*}(\bar{p}, \overline{\boldsymbol{p}}), D^{B}(\bar{p}, \overline{\boldsymbol{p}})\right]\right]-c q^{*}(\bar{p}, \overline{\boldsymbol{p}})$, which we denote as $\pi^{*}(\bar{p}, \overline{\boldsymbol{p}})$. However, because $q^{*}$ is determined in a newsvendor fashion before demand $D^{B}$ is realized, there is a nonzero probability that some inventory is left over. By deviating to $\bar{p}-\varepsilon$, yet ordering $q^{*}(\bar{p}, \overline{\boldsymbol{p}})$, firm $i$ increases its expected profit because with $\varepsilon \downarrow 0$ :

$$
\begin{aligned}
& (\bar{p}-\varepsilon) E\left[\min \left[q^{*}(\bar{p}, \overline{\boldsymbol{p}}), D^{B}(\bar{p}-\varepsilon, \overline{\boldsymbol{p}})\right]\right]-c q^{*}(\bar{p}, \overline{\boldsymbol{p}}) \rightarrow \\
& \bar{p} E\left[\min \left[q^{*}(\bar{p}, \overline{\boldsymbol{p}}), D^{B}(\bar{p}-\varepsilon, \overline{\boldsymbol{p}})\right]\right]-c q^{*}(\bar{p}, \overline{\boldsymbol{p}})> \\
& \bar{p} E\left[\min \left[q^{*}(\bar{p}, \overline{\boldsymbol{p}}), D^{B}(\bar{p}, \overline{\boldsymbol{p}})\right]\right]-c q^{*}(\bar{p}, \overline{\boldsymbol{p}})=\pi^{*}(\bar{p}, \overline{\boldsymbol{p}}),
\end{aligned}
$$

where the inequality follows from property [2]. That is, at any $\bar{p}>p_{L}$, firm $i$ has an incentive to undercut the price, which implies that the only equilibrium price for such $\{k\}$ firms is $\bar{p}=p_{L}$.

Summarizing, despite having a flexibility to charge any price, in equilibrium, each firm will either charge $p_{H}$ per the FOC above, or $p_{L}$, which is the exogenous lower-bound on the item price. The only clarification we make is that, in Section 5, the two [exogenous] prices, $p_{H}, p_{L}$ and the range of "allowable" demand "elasticity", $\mu$, is endogenously restricted to $\mu \in\left[0, \frac{\lambda\left(p_{H}-p_{L}\right)}{p_{H}-c}\right]$. This restriction ensures that $p_{H}$ is indeed the better of the two prices to charge should a firm decide to focus on the loyal customers only; it is easy to see that the upper bound on $\mu$ comes from $\left(p_{H}-c\right) \frac{\lambda-\mu}{N}>\left(p_{L}-c\right) \frac{\lambda}{N} \Leftrightarrow \mu<$ $\frac{\lambda\left(p_{H}-p_{L}\right)}{p_{H}-c}$. With the linear demand function considered in Section 3, however, the starting points are the [exogenous] lower bound on the prices, $p_{L}$ and an elasticity-like parameter $\mu \geq 0$, which influence the intercept and slope of the demand function, and the FOC from which $p_{H}$ is derived endogenously. This way, $p_{H}$ is automatically better than $p_{L}$ for a firm that only targets the loyal consumers, and hence no condition (on $\mu$, or, equivalently, $a$ and $b$ ) is necessary.

## Appendix B.3: $E[p]$ when it is defined as the average over customers

We defined $\boldsymbol{E}[\boldsymbol{p}]$ by taking the average over the firms ("average price charged"). In this appendix, we show that all the results hold when the average is taken over customers ("average price paid"). The number of customers that pay the low price is $\frac{\lambda}{N} k^{*}+1-\lambda$, and the number of customers that pay the
high price is $\frac{\lambda-\mu}{N}\left(N-k^{*}\right)$, so the average price that customers pay is $E[p]_{\text {cust }}=$ $\frac{\frac{\lambda-\mu}{N}\left(N-k^{*}\right) p_{H}+\left(\frac{\lambda}{N} k^{*}+1-\lambda\right) p_{L}}{\frac{\lambda-\mu}{N}\left(N-k^{*}\right)+\left(\frac{\lambda}{N} \kappa^{*}+1-\lambda\right)}$.

The conditions in Propositions 1a and 1b stem from the defection $k^{*}$ do (cf. Equation A3), and thus are unaffected by how we define the average price. At the same time $E[p]_{\text {cust }}$ increases in $N$ if $k^{*}$ does not change because $\quad \frac{\frac{\lambda-\mu}{N+1}\left(N+1-k^{*}\right) p_{H}+\left(k^{*} \frac{\lambda}{N+1}+1-\lambda\right) p_{L}}{\frac{\lambda-\mu}{N+1}\left(N+1-k^{*}\right)+\left(k^{*} \frac{\lambda}{N+1}+1-\lambda\right)}-\frac{\frac{\lambda-\mu}{N}\left(N-k^{*}\right) p_{H}+\left(k^{*} \frac{\lambda}{N}+1-\lambda\right) p_{L}}{\frac{\lambda-\mu}{N}\left(N-k^{*}\right)+\left(k^{*} \frac{\lambda}{N}+1-\lambda\right)}=$ $\frac{k^{*}\left(p_{H}+p_{L}\right)(\lambda-\mu)}{(N(1-\mu)+k \mu)((1+N)(1-\mu)+k \mu)}>0$. Similarly, $E[p]_{\text {cust }}$ decreases in $N$ if $k^{*}$ increases by one because $\frac{\frac{\lambda-\mu}{N+1}\left(N+1-\left(k^{*}+1\right)\right) p_{H}+\left(\left(k^{*}+1\right) \frac{\lambda}{N+1}+1-\lambda\right) p_{L}}{\frac{\lambda \mu}{N+1}\left(N+1-\left(k^{*}+1\right)\right)+\left(\left(k^{*}+1\right) \frac{\lambda}{N+1}+1-\lambda\right)}-\frac{\frac{\lambda-\mu}{N}\left(N-k^{*}\right) p_{H}+\left(k^{*} \frac{\lambda}{N}+1-\lambda\right) p_{L}}{\frac{\lambda-\mu}{N}\left(N-k^{*}\right)+\left(k^{*} \frac{\lambda}{N}+1-\lambda\right)}=\frac{-\left(N-k^{*}\right)\left(p_{H}+p_{L}\right)(\lambda-\mu)}{(N(1-\mu)+k \mu)(1+N(1-\mu)+k \mu)}<0$.

Combining the two results, the dependencies outlined in Propositions 1a and 1b (and consequently in Proposition 2a,) continue to hold.

Propositions 1c and 2b become $\lim _{N \rightarrow \infty} E[p]_{\text {cust }}=\frac{\lambda-\mu}{1-\mu} p_{H}+\frac{1-\lambda}{1-\mu} p_{L}$ and $\lim _{N \rightarrow \infty} E\left[p_{B}\right]_{\text {cust }}=p_{H}-$ $\frac{\left(p_{H}-c\right)(1-\lambda)\left(p_{H}-p_{L}\right)}{p_{H}-p_{L}-\mu\left(p_{H}-c\right)}$.

Proposition 2 c (i.e., that $E[p]_{\text {cust }} \geq E\left[p_{B}\right]_{\text {cust }}$ ) holds because $E[p]_{\text {cust }}-E\left[p_{B}\right]_{\text {cust }}=$ $\frac{\left(k_{B}^{*}-k^{*}\right) N\left(p_{H}-p_{L}\right)(\lambda-\mu)}{\left(N(1-\mu)+k^{*} \mu\right)\left(N(1-\mu)+k_{B}^{*} \mu\right)} \geq 0$.

Below are the instructions that the students who played the Supply Chain game received.

## The Supply Simulation Chain Game

## Introduction

The purpose of the supply chain game is to expose participants to the difficulties of making decisions on orders within a supply chain context and to explore the dynamic behavior of supply chains in which individual firms make decisions independently. You will manage a global supply chain that competes by making timely decisions relating to pricing, ordering, and logistics as well as supply chain contracts and see your impact on the market and their competitors.

The game is played in teams of between 4 to 7 players. A team represents a supply chain in which there are 4 'roles': Retailer, Wholesaler, Distributor, and Manufacturer. Thus, each role will be played by one or two people (Exhibit 1 describes the supply chain flow). The retailer sells the product to consumers and receives it from the Wholesaler. The Wholesaler sells the product to the Retailer and receives it from the Distributor and so forth.


Exhibit 1: The Supply Chain Flow
In each round, each player receives an order from further down the supply chain (for the Retailer, the demand will be customer demand, for the Wholesaler it will be Retailer orders and so on) and fills that order (if possible). Players also decide the size of the order to place with their suppliers. There is a lead time of two weeks (see more on that below) and an order delay of one week (that is when you order this week your supplier will receive that order at the beginning of next week), so what is ordered in one week does not arrive till three weeks later. Any unmet demand is backlogged and will be met as soon as possible in the future.

The teams compete to manage their supply chain successfully, while maximizing their own profit. Your object as a team is to provide the best customer service at the lowest cost. Each time your team provides poor customer service, your team will be charged a penalty cost (backlog cost). You can provide better
service by carrying more inventory in your supply chain, but the more inventory you carry, the more holding costs you will be charged. (Details of how costs are charged are given below.) Each player is measured against their counterpart in other supply chains and thus the retailer's profit is measured compared to the other retailers, the wholesaler's profit compared to the other wholesalers, etc.

## Game Description

## The product and market

The multiple supply chains in the market compete on price to gain market share. There are two types of customers: $40-50 \%$ of the customers are non-loyal customers that will buy the product at the lowest price regardless of the brand. The rest of the customers are divided equally between all the supply chains and are brand loyal and price sensitive. Experts expect the total market size for the product to reach between 1 and 2 million in (unit) sales in the first year.

## The supply chain contract

The supply chain contract signed by each team determines the transfer payments among the supply chain members. At the beginning of the game and for the first six months the players can only use a wholesale-pricing-contract where each member decides on their margin and thus the wholesale price they will charge their downstream customer. That is, the manufacturer decides based on the production cost for the product, how much to charge the distributor. The distributor, based on the wholesale price charged by the manufacturer, decides how much to charge the wholesaler, and so on. After the first six months, the supply chain members can use revenue sharing in their contract and thus the contract is based on two parameters: the revenue each member receives from the retailer and the wholesale price charged from their downstream customer.

## The Decisions

## Beginning of the game

At the beginning of the game there is one major decision the players need to make. Each player needs to decide about the wholesale price they would charge their downstream customers as part of the wholesale-pricing-contract each supply chain use for the first six months (since it is not possible to use revenue sharing at this stage). The retailer will have to decide the retail price to charge in the first quarter, although this will be entered only in week 1 and not as part of the contract pre-game stage.

## Weekly plays

In each week, each player receives an order from further down the supply chain and fills that order (if possible). Players make two decisions, first what is the size of the order to place with their suppliers (for the manufacturer this is the amount to produce), and second, how fast would they like to receive this order. There is a lead time of two weeks for normal delivery and of one week for fast delivery and no order delay (that is when you order this week your supplier receives the order immediately and will ship it based on availability and your shipping choice).

## Quarterly Decisions

At the beginning of each quarter (12 weeks) after receiving market information, the retailer can decide on the retail price to charge the end customers. From timing perspective, this means that the retail price will be entered in week 1 , week 13 , week 25 , etc.

## Half-Year Meetings

At the end of every half a year ( 24 weeks) each team will meet to decide on their supply chain and pricing strategy. In this meeting the team will be able to receive market information for the last two quarters and discuss their decisions and business success in these two quarters as well as the performance and strategies of the other brands in the market. The team will also be able to change its current contract.

## The Costs

## Inventory holding cost

Each player pays a carrying cost of $0.5 \%$ of the buy price (their cost) per unit per week for carrying Inventory. For example, if the wholesale price charged by the wholesaler to the retailer was $\$ 200$, then the inventory holding cost for the retailer is $\$ 1(0.5 \% * \$ 200)$ per unit per week. This is the cost of providing good customer service.

## Backlog penalty cost

Each player pays a penalty cost of $1 \%$ of their sales price per unit per week for not being able to meet their demand. For example, if the retail price is $\$ 300$, then the backlog penalty cost for the retailer is $\$ 3$ $(1 \% * \$ 300)$ per unit per week. This is the cost of poor customer service. It could be seen as a discount the retailer is forced to give his customers for waiting for the product one more week or the loss of goodwill caused by the delay.

## Transportation cost

Each player pays a transportation cost of $\$ 2$ per unit for normal delivery that will transfer the units by truck and a cost of $\$ 4$ per unit for expedited delivery that will transfer the units by plane (fast mode). As described above, there is a lead time of two weeks for normal delivery and of one week for expedited delivery.

## Production cost

The production cost of the product for the manufacturer is between $\$ 5$ and $\$ 35$, where the actual cost value is known only to the manufacturer.

## The starting position

## Inventory

As part of the planning for the start of the selling season, at the beginning of the game, each player holds 20,000 units in inventory. Thus, each supply chain holds a total of 80,000 units of finished goods inventory.

The mechanics of how the game is played as well as the interface and the way to make decisions are described in the next page.

## The Game Interface

## Starting the game

Before the game starts each player will receive a password based on their role and team number. Using this password you can login to the game using the next screen:


After clicking login you will be moved to the next screen where you can provide your name and click on "Start Game".


## The Contract

As soon as all the players in your team clicked on "Start Game" you will be transferred to the contract screen, where you can enter your contract parameters. In the beginning of the game, the only parameter will be the wholesale price, however, in later stages revenue sharing is allowed. For example, as the distributor after the first six months, you will have to decide your wholesale price based on the wholesale price charges by the manufacturer and the revenue share you receive from the retailer. You will then need to enter your price and click "Submit Offer".


As soon as all team members submit their offer the next screen appears. If all contract terms are agreeable, the team members should accept the contract.


## The weekly game interface

As soon as all the players accepted the contract, the game will begin with the next screen at week 1 . The screen has three main parts:

1) (a) Current week information: This includes the previous week's demand, the current inventory, and the current units on order
(b) Current week decisions: This includes the order quantity decision and the mode of transportation decision for each player. At the beginning of each quarter this also includes the retail price decision for the retailer
2) Inbound shipment information: This provides graphical information about all shipments that were sent from the direct supplier and are on their way to the player as well as the mode of transportation used
3) Summary table information: The table summarizes information such as demand, inventory level, revenues, inventory holding cost, backlog penalty cost, and profits for each week up to the previous week


[^0]:    ${ }^{1}$ The game also has several additional, more complex decisions made by the teams, such as what supply chain contract to use, made every two quarters, however, they are not the focus of this paper.

[^1]:    ${ }^{2}$ https://www.quora.com/Is-Target-a-more-premium-big-box-store-than-Walmart-or-do-they-compete-for-the-same-demographic-of-customers accessed on June 4, 2018.
    ${ }^{3}$ Canadian grocery retail has extremely strong local players, and Walmart is not a significant player.

[^2]:    ${ }^{4}$ We consider a single ordering decision per firm; hence, the order quantity, capacity, and inventory are identical.
    ${ }^{5}$ Because firms order before demand is realized, they are like the price-setting newsvendors, Petruzzi and Dada (1999). However, unlike much of the newsvendor competition literature, our firms compete on price and not on inventory (i.e., for the initial and not for the overflow demand). Unsatisfied / overflow demand in our model is lost, same as in Chen et al. (2004). Therefore, to avoid potentially misleading associations we do not refer to our firms as "newsvendors." We thank an anonymous reviewer for pointing this out.

[^3]:    ${ }^{6}$ We also considered multiplicative demand uncertainty, $\varepsilon \times\left(a-b p_{i}\right)$, for $\varepsilon \in \operatorname{Uniform}[1-B ; 1+B]$. We tested multiple values of $B$ and found that the solutions hardly change; the results presented here are for $B=0.05$.

[^4]:    ${ }^{7}$ For example, at $p_{i}=2$, with $N=2$, the loyal demand is $\frac{\lambda}{N}\left(a-b p_{i}\right)=0.5 / 2 \times(500-100 \times 2)=75$, while the loyal+bargain-hunters' demand is $\frac{\lambda}{N}\left(a-b p_{i}\right)+(1-\lambda)\left(a-b p_{i}\right)=75+(1-0.5) \times(500-100 \times 2)=$ 225, - a $3 X$ difference. With $N=4$ these numbers become 37.5 and 187.5 , respectively, a $5 X$ difference.

[^5]:    ${ }^{8}$ An extended model of bargain-hunting demand allocation presented in Appendix B. 1 also offers a more nuanced understanding of over-stocking and stockouts: in particular, when the fraction of bargain-hunting demand won by a single low-priced firm is below a certain threshold, such a firm may experience a stockout in equilibrium.
    ${ }^{9}$ An alternative approach would be to take the expectation not over the firms who sell the product (as we did above), but rather over the consumers who buy it. This will make the expressions for the average prices slightly different, but our qualitative results would still hold; see Appendix B.3.
    ${ }^{10}$ Settings where the average price increases monotonically also exist, e.g., $p_{H}=10, p_{L}=8, c=5, \lambda=0.7, \mu=$ 0.05 .

[^6]:    ${ }^{11}$ The second condition guarantees $k_{B}^{*} \leq N-1$. The condition is satisfied when the production cost, $c$, is sufficiently large, or when the prices $p_{H}$ and $p_{L}$ are sufficiently small, i.e., when there is a certain minimum "amount" of inventory risk. Otherwise, because there is less inventory risk under the MTO system to begin with, all firms would charge a low price in equilibrium.

[^7]:    ${ }^{12}$ Technically, this argument, and Proposition 3 in general, hold only in the limit, but the numerical illustration in Figure 5 shows that the intuition is true in most cases, even when $N$ is small.

